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Axially Symmetric Incompressible Turbulent Wake Downstream of a Single Body

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Nomenclature

- B = constant
- b = half-width of wake
- C = constant
- C_D = drag coefficient
- D = drag
- d = diameter of body
- l = mixing length
- u = streamwise velocity component
- u_1 = $u_1 = u_\infty - u$
- u_∞ = freestream velocity
- v = velocity component parallel to y coordinate
- x = streamwise coordinate
- y = coordinate perpendicular to x
- β = constant
- η = $\eta = y/b$
- ρ = density of fluid
- τ = shear stress

Introduction

The spread and stream velocity distribution of axially symmetric incompressible turbulent wake (Fig. 1) downstream of a body of revolution are determined. The uniformity of static pressure and similarity of the velocity profile are assumed, and, for the analysis, Prandtl's mixing length theory is applied. The obtained wake solution is an extension of the known two-dimensional analysis.

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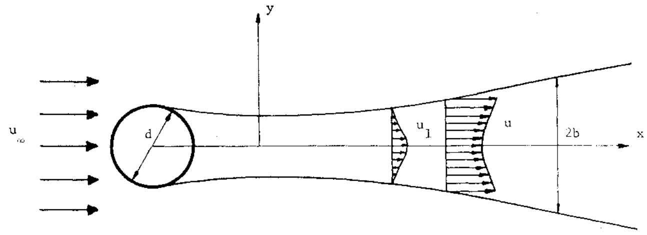

Fig. 1 Incompressible turbulent wake.

Wake Flow Analysis

The sketch shows the coordinate system and the velocity distribution. The governing equations are

Continuity:

$$[\partial(yu)/\partial x] + [\partial(yv)/\partial y] = 0$$

Momentum:

$$u(\partial u/\partial x) + v(\partial u/\partial y) = (1/y\rho)[\partial(y\tau)/\partial y]$$

In similar velocity profiles, the velocity difference u_1 defined by

$$u_1 = u_\infty - u$$

is small compared to u_∞. By substituting u_1 into these two equations and considering the order of magnitude, the momentum equation is reduced to

$$-u_\infty(\partial u_1/\partial x) = (\tau/y\rho) + (1/\rho)(\partial\tau/\partial y)$$

Now introducing Prandtl's mixing length l,

$$\tau = \rho l^2|\partial u/\partial y|\partial u/\partial y$$

we obtain

$$-u_\infty \frac{\partial u_1}{\partial x} = \frac{l^2}{y}\left(\frac{\partial u_1}{\partial y}\right)^2 + 2l^2 \frac{\partial u_1}{\partial y}\frac{\partial^2 u_1}{\partial y^2} \tag{1}$$

Since for an axially symmetric turbulent incompressible wake, b and u_1 are proportional to $x^{1/3}$ and $x^{-2/3}$, respectively,[1] it is assumed that b and u_1 are given by

$$b = B(C_D d^2 x)^{1/3} \quad \text{and} \quad \frac{u_1}{u_\infty} = \left(\frac{x^2}{C_D d^2}\right)^{-1/3} f(\eta)$$

The constant B is to be determined later.

Relating β by $l = \beta b$, Eq. (1) is reduced to

$$\tfrac{1}{3}(2f + \eta f') = 2(\beta^2/B)[f'f'' + (1/2\eta)f'^2]$$

The solution of this differential equation is

$$\tfrac{1}{3}\eta f = (\beta^2/B)f'^2 + C_1 \tag{2}$$

where $'$ denotes differentiation with respect to η.

From the boundary conditions at $\eta = 1, f = f' = 0$, it follows that $C_1 = 0$. Equation (2) then leads to

$$df/(f)^{1/2} = (B/3\beta^2)^{1/2}(\eta)^{1/2}d\eta$$

Integrating, we obtain

$$2(f)^{1/2} = \tfrac{2}{3}(B/3\beta^2)^{1/2}\eta^{3/2} + C_2$$

From the boundary condition at $\eta = 1, f = 0$, it follows that

$$C_2 = -\tfrac{2}{3}(B/3\beta^2)^{1/2}$$

Hence

$$f = \tfrac{1}{3}(B/9\beta^2)(1 - \eta^{3/2})^2$$

This result is similar to that of two-dimensional wake flow obtained by Schlichting,[1] differing only in the coefficient from $\frac{1}{2}$ to $\frac{1}{3}$, as also indicated in Eq. (2).

In order to determine B, apply the momentum theorem to a control volume enclosing the body. The total drag is given by

$$D = 2\pi\rho \int_0^\infty u(u_\infty - u)ydy$$

$$= 2\pi\rho \int_0^\infty u_1(u_\infty - u_1)ydy$$

Neglecting u_1^2 and defining

$$D = (\tfrac{1}{2})C_D\rho u_\infty^2(\pi d^2/4)$$

we obtain, by equating these two equations,

$$\left(\frac{1}{16}\right) u_\infty C_D d^2 = \int_0^\infty u_1 ydy$$

$$= \int_0^1 u_1 b^2 \eta d\eta$$

$$= u_\infty C_D d^2 \frac{B^3}{27\beta^2} \int_0^1 (\eta^4 - 2\eta^{5/2} + \eta)d\eta$$

Thus

$$B = [(105)^{1/3}/2]\,\beta^{2/3}$$

Therefore,

$$f = [(105)^{1/3}/54]\beta^{-4/3}(1 - \eta^{3/2})^2 \tag{3}$$

Finally, using Eq. (3),

$$b = [(105)^{1/3}/2]\beta^{2/3}(C_D d^2 x)^{1/3} \tag{4}$$

and

$$u_1/u_\infty = [(105)^{1/3}/54]\beta^{-4/3}\,(x^2/C_D d^2)^{-1/3}[1 - (y/b)^{3/2}]^2 \tag{5}$$

Concluding Remarks

The solutions given by Eqs. (4) and (5) are more direct and simpler than Swain's analysis[2] of the axial-symmetric wake flow.

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Announcement: 1968 Author and Subject Indexes

It has been the custom to publish the annual author and subject indexes of the AIAA journals in the last issue of the year. This year, however, with the approval of the Publications Committee, we will publish a combined index of the four journals (*AIAA Journal, Journal of Spacecraft and Rockets, Journal of Aircraft, and Journal of Hydronautics*). All topic headings will be included, whether or not anything on that subject was published. The index will be mailed to all subscribers to the journals in January 1969. We hope that readers will find the combined index more convenient to use than four separate ones.

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